

Investigation of the dynamical structure factor of the Nagel-Schreckenberg traffic flow model

S. Lübeck, L. Roters, and K. D. Usadel

Theoretische Physik, Gerhard-Mercator-Universität, 47048 Duisburg, Deutschland

Abstract. The Nagel-Schreckenberg traffic flow model shows a transition from a free flow regime to a jammed regime for increasing car density. The measurement of the dynamical structure factor offers the chance to observe the evolution of jams without the necessity to define a car to be jammed or not. Above the jamming transition the dynamical structure factor $S(k, \omega)$ exhibits for a given k -value two maxima corresponding to the separation of the system into the free flow phase and jammed phase. Analyzing the k -dependence of these maxima the backward velocity of the jams is measured. We find that the jam velocity neither depends on the global density of the cars nor the maximal velocity of the model.

1 Introduction

In 1992 Nagel and Schreckenberg [1] introduced a simple cellular automata model, which simulates single-lane one-way traffic, in order to study the transition from free flow traffic to jammed traffic with increasing car density. The behavior of the model is determined by three parameters, the maximal velocity v_{\max} , the noise parameter P and the global density of cars $\rho = N/L$, where N denotes the total number of the cars and L the system size, respectively. The variables describing a car i at time t are its position $r_i \in \{1, 2, \dots, L\}$, its velocity $v_i \in \{0, 1, \dots, v_{\max}\}$ and the gap g_i , which is the number of unoccupied cells in front of the car. Using periodic boundary conditions, the following update steps are applied in parallel for each car:

$$\begin{aligned} v_i &\longmapsto \min(v_{\max}, v_i + 1), \\ v_i &\longmapsto \min(v_i, g_i), \\ v_i &\longmapsto \max(0, v_i - 1) \quad \text{with probability } P, \\ r_i &\longmapsto r_i + v_i. \end{aligned}$$

Increasing the global density for fixed P and v_{\max} jams occur above a certain critical value ρ_c [1]. Contrary to the forward movement of all particles the jammed region is characterized by a backward moving of shock waves, i.e. jams are nothing else than backward moving density fluctuations. This property of jams was found in real traffic flow already in the 50's [2]. Investigations of the Nagel-Schreckenberg traffic flow model show that crossing the critical point a transition takes place from a homogeneous regime (free flow phase) to an inhomogeneous regime which is characterized by a coexistence of two phases (free

flow and jammed traffic). Thereby, the free flow is characterized by a low local density and the jammed phase by a high local density, respectively. Due to the particle conservation of the model the transition is realized by the system separating into a low density region and a high density region [3]. Therefore, the dynamical structure factor is an appropriate tool to investigate the decomposition of the two phases above the transition point. Additionally the dynamical structure factor has the decisive advantage that both phases could be distinguished in a natural way by the sign of their characteristic velocities. Thus no artificial definitions of jams are needed which were used in previous investigations and lead to controversial results (see for instance [3,4,5,6]).

2 Simulation and Results

The dynamical structure factor is defined as follows: Let

$$\eta_{r,t} = \begin{cases} 1 & \text{if cell } r \text{ is occupied at time } t \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

The evolution of $\eta_{r,t}$ leads directly to the space-time diagram where the propagation of the particles can be visualized. Figure 1 shows a space-time diagram of the system above the critical value. Traffic jams are characterized by the backward movement of the high density regions. The dynamical structure factor $S(k, \omega)$ is given by

$$S(k, \omega) = lT \left\langle \left| \frac{1}{lT} \sum_{r,t} \eta_{r,t} e^{i(kr - \omega t)} \right|^2 \right\rangle, \quad (2)$$

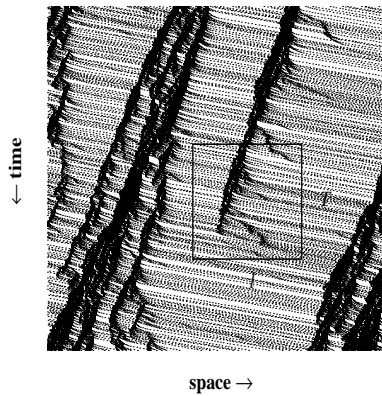


Fig. 1. Space-time plot for $v_{\max} = 5$, $P = 0.5$, and $\rho > \rho_c$. Note the separation of the system in high and low density regions.

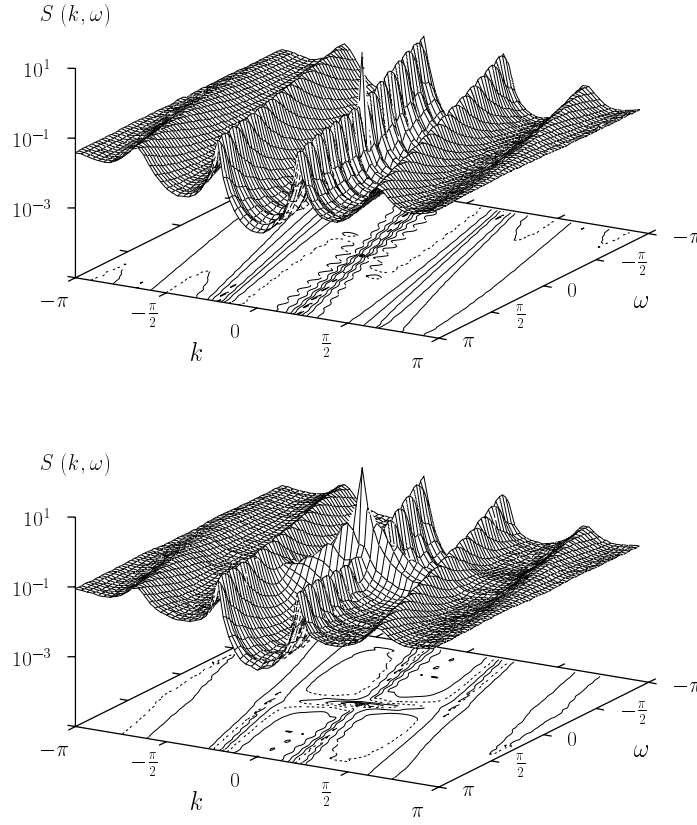


Fig. 2. The dynamical structure factor $S(k, \omega)$ below (upper figure) and above (lower figure) the critical value. The ridges with maximal $S(k, \omega)$ indicates the various modes.

where the Fourier transform is taken over a finite rectangle of the space-time diagram of size $l \times T$ (see Fig. 1).

In Fig. 2 we plot the dynamical structure factor below and above the transition, respectively. Below the transition $S(k, \omega)$ exhibits one mode formed by the ridges. The values of the modes are obtained from a determination of the position of the maxima of $S(k, \omega)$ for fixed ω or k , respectively. This mode is characterized by a positive slope $v = \frac{\partial \omega}{\partial k}$ corresponding to the positive velocity of the particles in the free flow phase. Increasing the global density of the cars, a second mode appears at the transition to the coexistence regime. This second mode exhibits a negative slope indicating that it corresponds to the backward moving density fluctuations in the jammed phase.

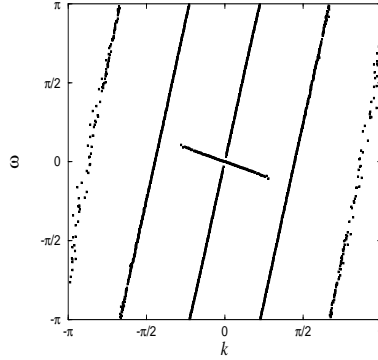


Fig. 3. The maximal values of the dynamical structure factor for $v_{\max} = 5$, $P = 0.5$, and $\rho > \rho_c$. The lines with positive slope corresponds to the free flow phase and the negative slope corresponds to the jammed phase, respectively.

In Fig. 3 we plot the position of the ridges of $S(k, \omega)$ in the jammed regime. Both modes are characterized by a linear dispersion relation ($\frac{\partial \omega}{\partial k} = \text{const}$), i.e. each phase is characterized by one velocity only. Since the modes of both phases can clearly be distinguished (note the logarithmic scale in Fig. 2) and no other mode occur we think that this indicates that above the transition a real phase separation of the system takes place. This is in contrast to results of previous investigations which examine the steady state correlation function and concluded that there is no real phase separation of the system [6].

A linear regression of the modes of Fig. 3 yields the velocities of both phases. In Fig. 4 we plot the velocities of the free flow and jammed phase for different values of P , v_{\max} , and ρ . Independent of the value of the global density, the free flow phase is characterized by the velocity $v_f = v_{\max} - P$, i.e. v_f equals the average velocity of the particles in the limit $\rho \rightarrow 0$ and the cars can be considered as independent particles. This agrees with investigations of the local density distribution and the steady state structure factor ($\omega = 0$) [3]. The plotted values of the jam velocity v_j are obtained from simulations with $P = 0.5$ and various values of ρ and v_{\max} . The velocity of the jams neither depends on the global density nor on the maximum velocity. The jam velocity is a function of the noise parameter only. Using the structure of $S(k, \omega)$ we are able to determine this P dependence of v_j for the first time and the results are plotted in Fig. 5.

3 Conclusions

We studied numerically the Nagel-Schreckenberg traffic flow model. The investigation of the dynamical structure factor allowed us to examine the transition of the system from a free flow regime to a jammed regime. Above the transition the dynamical structure factor exhibits two modes corresponding to the coexisting

free flow and jammed phase. Due to the sign of their characteristic velocities v_f and v_j , both phases can clearly be distinguished. We investigated the dependence of v_f and v_j on the system parameters systematically and found that the jammed velocity depends on the noise parameter only. The characteristic velocity of the free flow phase turned out to be equal to the velocity of free flowing cars in the low density limit ($\rho \rightarrow 0$), i.e. cars of the free flow phase behave as independent particles.

References

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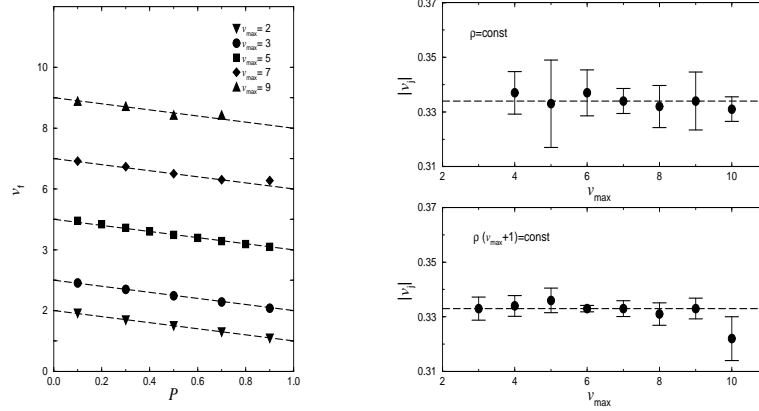


Fig. 4. The characteristic velocities of the free flow phase v_f (left) and jammed phase v_j (right) for various values of the parameters P , v_{\max} , and ρ . The dashed lines in the left figure correspond to the average velocity $v_{\max} - P$ of a single particle. The values of v_j are obtained from simulations with $P = 0.5$ and various values of ρ and v_{\max} .

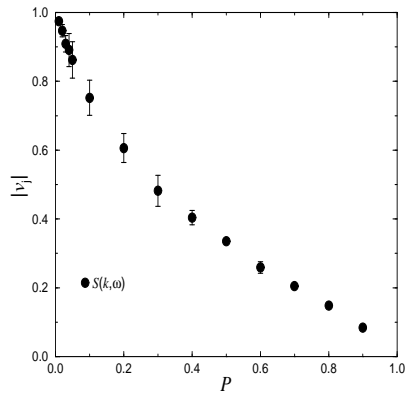


Fig. 5. The jam velocity v_j as a function of the noise parameter P .